PHIL 4310: Advanced Logic
Spring 2026
Homework 1

Read Chapter 1 of *Logic for Philosophy*

If you are interested in infinity, I recommend chapter 3 of Joel David Hamkins' *Lectures on the Philosophy of Mathematics*.

Homework questions:

Part I:

1) In the story of Hilbert's hotel, the goal is to accommodate a group of guests that arrive. Are there any circumstances where it would matter whether the hotel starts out empty vs. full? That is, where it is possible to accommodate the new guests if the hotel starts out empty, but not possible if it starts out full? Explain your answer.

2) FACT: The set of all sets of positive integers is uncountable. PROOF: For reductio, assume that we have a list of all such sets S_1, S_2, S_3, etc. Now construct a set not on this list using diagonalization.

-- Carefully finish this proof explaining each step.

3) Prove that the set of all finite sets of positive integers is countable. (HINT: To prove this, produce an ordered list of all such sets). Why can we not use the diagonalization technique from the previous problem to show that that this set is uncountable?

4) FACT: Every open interval on the real line is equinumerous with any other. It might help to see some easy examples. For example, (0,1) is equinumerous with (0,2). Here is a bijection: f(x) = 2x. (0,1) is also equinumerous with (-1,2) - consider f(x) = 3x - 1. Now give a bijection to show that (a,b) is equinumerous with (c,d).

5) Prove that the open interval (0,1) is also equinumerous with the entire real line. HINT: consider the graph of f(x) = tan(x) when x is in (-π/2, π/2).

6) Now that you know that there are bijections between the natural numbers and the even numbers and between the points on a line segment and the points in a larger dimension space like $\mathbb{R}^n$, but not between the integers and the real numbers, do you think that the claim that "One set is the same size as another set if and only if there is a 1:1 correspondence between their members" is a good definition of "size"? Do you think that the question of what "size" means in this context is just a convention or stipulation? Is it a question of logic? Of mathematics? A philosophical question? Or what?

*Note: Problems 3-5 above are more mathematical in nature. If you want to skip those problems, fine, do three extra problems from exercise 2.1 below.

Part II:

Read Chapter 2 - through section 2.2.3 (through page 52) of *Logic for Philosophy*

DO: Exercise 2.1 b, f, h, l; 2.2 a-d
Note - you should be able to do all of the problems in these exercises if need be.

Also do:

Multiple choice: Pair up each sentence on the first list with an English translation on the second list. (This is not necessarily a 1-1 pairing.)

First list:

1) $A \vee B \vee C$
2) $\sim(A \wedge B) \wedge \sim(A \wedge C) \wedge \sim(B \wedge C)$
3) $(A \wedge \sim B \wedge \sim C) \vee (\sim A \wedge B \wedge \sim C) \vee (\sim A \wedge \sim B \wedge C)$
4) $(A \wedge B) \vee (A \wedge C) \vee (B \wedge C)$
5) $(A \vee B) \wedge (A \vee C) \wedge (B \vee C)$
6) $(A \wedge B \wedge \sim C) \vee (A \wedge C \wedge \sim B) \vee (B \wedge C \wedge \sim A)$
7) $(A \vee B \vee C) \rightarrow (A \wedge B \wedge C)$
8) $(A \rightarrow B) \wedge (B \rightarrow C) \wedge (C \rightarrow A)$
9) $A \leftrightarrow (B \leftrightarrow C)$
10) $((\sim A \wedge \sim B) \vee (\sim A \wedge \sim C) \vee (\sim B \wedge \sim C))$
11) $(A \rightarrow (\sim B \wedge \sim C)) \wedge (B \rightarrow (\sim A \wedge \sim C)) \wedge (C \rightarrow (\sim A \wedge \sim B))$
12) $(\sim A \rightarrow (B \wedge C)) \wedge (\sim B \rightarrow (A \wedge C)) \wedge (\sim C \rightarrow (A \wedge B))$
13) $(A \rightarrow (B \vee C)) \wedge (\sim A \rightarrow (B \wedge C))$
14) $A \leftrightarrow (B \leftrightarrow \sim C)$

Second list:

A) At least one of A, B, and C is true
B) At least two of A, B, and C is true
C) At most one of A, B, and C is true
D) At most two of A, B, and C is true
E) Exactly one of A, B, and C is true
F) Exactly two of A, B, and C is true
G) Either all or none of A, B, and C is true
H) Either exactly one or exactly two of A, B, C is true
I) None of A-H could be acceptably transcribed as the sentence in question

Part III:

In a certain place, all of the inhabitants are either knights or knaves. Knights always tell the truth and knaves always lie. You meet two inhabitants, A and B. A says, “B is a knave.” B says, “neither of us are knaves.” (Treat this as a single statement which is either true or false.) Determine whether A and B are knights or knaves (they might not be the same). Explain your reasoning. Be sure to explain why your answer *must* be right and not just that it is possibly right.